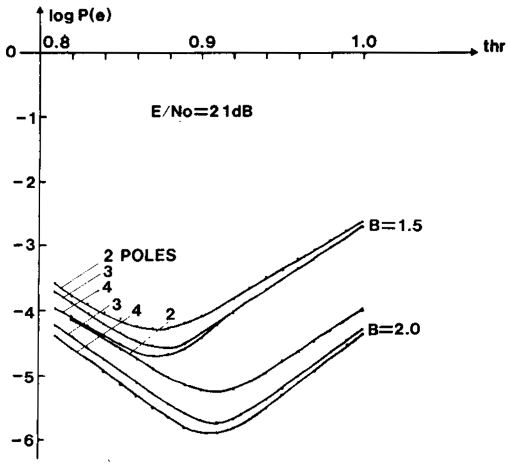

Fig. 5. Error probability as a function of threshold setting.

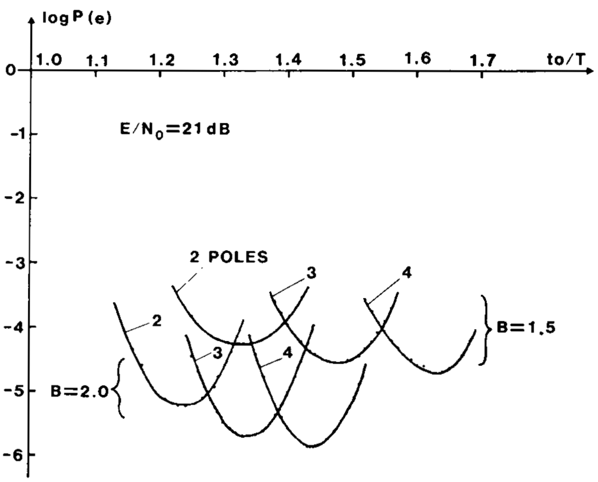

Fig. 6. Error probability as a function of sampling time.

see that (20) is overoptimistic for $E/N_0 > 21$ dB and overpessimistic for $E/N_0 < 21$ dB.

The optimization of t_0 and t_{hr} was performed by varying these parameters until a minimal value of $P(e)$ was obtained. Thus, in Figs. 5 and 6 we show the sensitivity of the error probability to the threshold setting and sampling time variation for $B = 1.5$ and 2.0.

In order to compare the quaternary and binary system, we have to restate Fig. 4 in terms of bit error probability and signal-to-noise ratio per bit. The binary case is actually already stated in these terms. For quaternary symbols the symbol energy E is related to the bit energy E_b by $E_b = E/2$; hence, relabeling the horizontal axis to E_b/N_0, all curves must be shifted to the left by 3 dB. If we use a Gray code to encode a pair of binary symbols into a quaternary symbol (for example: $-1-1 \leftrightarrow -3$, $-1+1 \leftrightarrow -1$, $+1+1 \leftrightarrow 1$, $+1-1 \leftrightarrow 3$), when symbol error occurs, usually only one of the bits will be in error; hence, the bit error probability $P_b(e)$ is related to the symbol error probability by $P_b(e) = P(e)/2$ and, relabeling the vertical axis to $\log P_b(e)$, all curves must be shifted downwards by $\log 2 = 0.3$. From the relabeled Fig. 4 we can establish that for a bit error probability of 10^{-6} and a normalized bandwidth of $B = 1.5$, quaternary symbols require 7 dB more energy per bit than binary symbols, and only 5.2 dB more energy when $B = 2.0$. This penalty in energy is compensated by the higher bit rate per bandwidth when quaternary symbols are used.

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Mean Packet Queueing Delay in a Buffered Two-User CSMA/CD System

HIDEAKI TAKAGI AND LEONARD KLEINROCK

Abstract—**We consider a system of two users of slotted CSMA-CD (carrier-sense multiple-access with collision detection). The two users are assumed to have independent identical packet arrival streams, the identical randomizing policy for retransmission, and an infinite capacity for storing queued packets. The mean packet delay (including the queueing and retransmission delays) is derived explicitly.**

We study the mean packet delay (which includes the queueing and randomized retransmission delays) in a finite population of users, each of whom has an independent packet arrival process and an infinite capacity for storing outstanding packets. When the channel access protocol is slotted ALOHA, this problem has been addressed in several papers. For example, Tobagi and Kleinrock [9] showed simulation results. Kleinrock and Yemini [2] developed a Wiener–Hopf technique in the case of two users. Saadawi and Ephremides [5] proposed an iterative approximation method using the notion of user and system Markov chains. Sidi and Segall [6] found an explicit expression for the mean delay in the case of two identical users. Approximations for the case of more than two users are also in [7] and [8].

In this correspondence we give an exact analysis leading to an explicit expression for the mean packet delay in the case of two identical users with slotted CSMA with collision detection, using the same technique as in [6]. We assume a constant packet length whose transmission time is chosen as the unit of time.

In CSMA we take into account the nonzero propagation delay, denoted by a, so that a successful transmission takes $1 + a$ units of channel time. We assume the collision detection to be such that an unsuccessful transmission lasts $b + a$,

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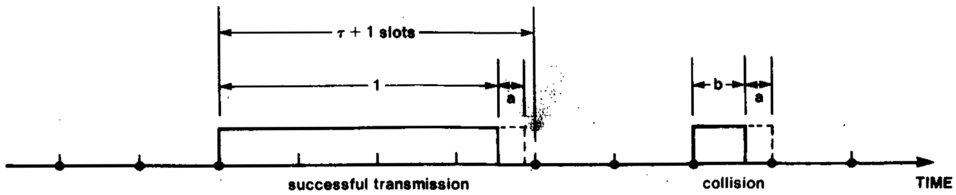

Fig. 1. An illustration of the channel state in slotted CSMA with collision detection. (The embedded Markov epochs are shown by •.)

where $a \le b \le 1$. Time is slotted with slot size equal to $b + a$, and the start of any transmission is synchronized with one of these slot boundaries.[1] At the end of every slot, each user can recognize what has happened in the slot. Clearly, an unsuccessful transmission takes up one slot and the duration of a channel idle period is also counted by slots. Let us define

$$\tau \triangleq \left\lceil \frac{1-b}{b+a} \right\rceil \tag{1}$$

where $\lceil x \rceil$ is the ceiling of x (let $\lceil i \rceil = i$ for an integer i). Then, a successful transmission takes $\tau + 1$ slots. See Fig. 1 for an illustration of successful and unsuccessful transmission periods. We note that the case without collision detection ($b = 1$) is equivalent to slotted ALOHA with slot size $1 + a$.

Consider two identical users with independent arrival processes and infinite buffers. Let λ and $f(z)$ be the mean and the generating function, respectively, for the number of arrivals at each user in any slot. Suppose that a user has at least one packet at the beginning of a given slot when he is not transmitting. If the preceding slot was sensed busy (due to the other user's transmission), he does not start transmission with probability 1. If the preceding slot was sensed idle (including the case where the preceding slot was an unsuccessful transmission or the last slot of a successful transmission), he starts transmission with probability p [and does not with probability ($\bar{p} = 1 - p$)], where $0 < p \le 1$. The simultaneous starts of transmission by both users result in an unsuccessful transmission. Otherwise the transmission will be successful, since its start is perceived in the first slot by the other user, who then suppresses his transmission.

Numbering the slot boundaries as $t = 1, 2, \cdots$, let $Q_i(t)$ ($i = 1, 2$) be the number of packets stored at user i at time t; this includes arrival(s) in the slot $[t-1, t]$ and excludes any packet that has successfully completed transmission in the slot $[t-1, t]$. From the above-mentioned arrival process and transmission protocol, it is clear that the process $[Q_1(t), Q_2(t)]$ is a (discrete-time) *semi-Markov process*. We can then construct an *embedded Markov chain* $[Q_1'(t'), Q_2'(t')]$, where $Q_i'(t')$ ($i = 1, 2$) is defined to be $Q_i(t')$ when t' is one of those slot boundaries which are not (properly) included in the transmission period. Obviously, these slot boundaries are the embedded Markov epochs in the sense that the process after t' depends on the state at t'. In Fig. 1, the Markov epochs are shown by •.

Now, we define the stationary joint generating function for the queue length distribution at the Markov epochs by

$$G(z_1, z_2) \triangleq \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \text{Prob } [Q_1' = k_1, Q_2' = k_2] z_1^{k_1} z_2^{k_2}. \tag{2}$$

[1] The main reason for using $b + a$ rather than a as the slot size is analytical tractability. Lam [3] used $2a$ as the slot size.

The equation for $G(z_1, z_2)$ is then given by

$$\begin{aligned} G(z_1, z_2) = F(z_1, z_2)\Big\{ & G(0, 0) \\ & + \left[\bar{p} + F(z_1, z_2)^{\tau} \frac{p}{z_1}\right][G(z_1, 0) - G(0, 0)] \\ & + \left[\bar{p} + F(z_1, z_2)^{\tau} \frac{p}{z_2}\right][G(0, z_2) - G(0, 0)] \\ & + \left[1 - 2p\bar{p} + F(z_1, z_2)^{\tau} p\bar{p}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)\right] \\ & \cdot [G(z_1, z_2) - G(z_1, 0) - G(0, z_2) + G(0, 0)]\Big\} \end{aligned} \tag{3}$$

where

$$F(z_1, z_2) \triangleq f(z_1) f(z_2). \tag{4}$$

Although we have been unable to solve (3) for $G(z_1, z_2)$, we can obtain the mean queue length $\bar{Q}_1 = \bar{Q}_2 = G_1(1, 1)$ as follows (following the approach in [6]). First, use the condition $G(1, 1) = 1$ and symmetry $G(1, 0) = G(0, 1)$ to get

$$p(1-2p)G(1, 0) + p^2 G(0, 0) = p\bar{p} - \frac{\lambda}{1 - 2\lambda\tau}. \tag{5}$$

Then, from (3), we can express $G_1(1, 1)$ and $dG(z, z)/dz|_{z=1}$ in terms of $G_1(1, 0)$ where $G_1(z_1, z_2) \triangleq dG(z_1, z_2)/dz_1$. By observation that $dG(z, z)/dz|_{z=1} = 2G_1(1, 1)$ due to symmetry, we find

$$G_1(1, 0) = \frac{2\lambda - \lambda^2 + f''(1)}{2p(1 - 2\lambda\tau)} \tag{6}$$

$$G_1(1, 1) = \frac{(1-2\lambda\tau)[f''(1)\bar{p} + 2\lambda\bar{p} + \lambda^2 p] - \lambda[2\lambda - \tau(\lambda^2 + f''(1)) - 2\tau(\tau+1)\lambda^2]}{2(1-2\lambda\tau)[p\bar{p}(1-2\lambda\tau) - \lambda]}. \tag{7}$$

where $f''(z) = d^2 f(z)/dz^2$.

Recall that $G_1(1, 1)$ is the mean queue length observed only at the embedded Markov epochs defined above. To find the mean queue length at an arbitrary slot boundary, note that the number of stored packets can be viewed as a "reward" in the context of the *semi-Markov process with reward* (see, e.g., [1]). Thus, if we denote by $l(k_1, k_2)$ the expected duration of the state (k_1, k_2), and by $b(k_1, k_2)$ the expected contribution to the backlog accumulation at state (k_1, k_2), then we have the total mean queue length at an arbitrary epoch (or the average reward) as

$$2\bar{Q} = \frac{B}{L} \tag{8}$$

where

$$L \triangleq \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} l(k_1, k_2) \text{ Prob } [Q_1' = k_1, Q_2' = k_2] \tag{9}$$

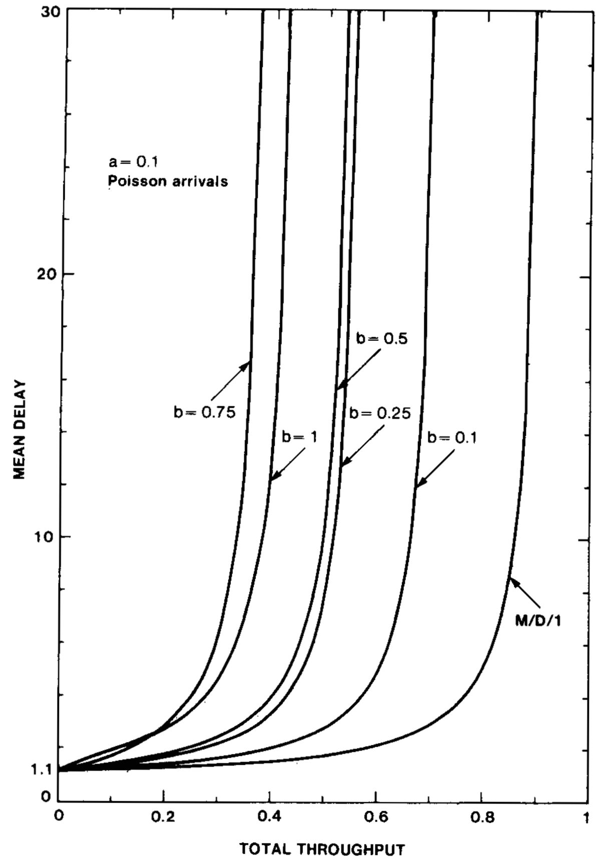

Fig. 2. Mean delay for two identical users of CSMA with collision detection.

and

$$B \triangleq \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} b(k_1, k_2) \text{ Prob } [Q_1' = k_1, Q_2' = k_2]. \quad (10)$$

Since the length between two successive Markov epochs is $\tau + 1$ when this interval involves a successful transmission, and it is 1 otherwise (either idle or unsuccessful transmission), we clearly have

$$l(k_1, k_2) = \begin{cases} 1 & k_1 = k_2 = 0 \\ p(\tau+1) + \bar{p} & k_1 \geq 1, \ k_2 = 0 \text{ or } k_1 = 0, \ k_2 \geq 1 \\ 2p\bar{p}(\tau+1) + 1 - 2p\bar{p} & k_1 \geq 1, \ k_2 \geq 1. \end{cases} \quad (11)$$

Similarly, the backlog accumulation is expressed as

$$b(k_1, k_2) = \begin{cases} 2\lambda & k_1 = k_2 = 0 \\ pB(k_1, \tau) + \bar{p}(k_1 + 2\lambda) & k_1 \geq 1, \ k_2 = 0 \\ pB(k_2, \tau) + \bar{p}(k_2 + 2\lambda) & k_1 = 0, \ k_2 \geq 1 \\ (1 - 2p\bar{p})(k_1 + k_2 + 2\lambda) + 2p\bar{p}B(k_1 + k_2, \tau) & k_1 \geq 1, \ k_2 \geq 1 \end{cases} \quad (12)$$

where $B(k, \tau)$ is the average accumulated backlog (in packets · slots) during a successful transmission period (of length $\tau + 1$ slots) which begins with k packets, and is given by

$$B(k, \tau) = (k-1)(\tau+1) + 1\left(-\left\lceil \frac{1+a}{b+a} \right\rceil + \frac{1+a}{b+a} + \tau\right) + \sum_{i=1}^{\tau+1} (2\lambda)i$$
$$= (k-1)(\tau+1) + \lambda(\tau+1)(\tau+2) + \frac{1-b}{b+a}. \quad (13)$$

Substituting (11)–(13) into (9) and (10), and making use of (5), we have the expressions for L and B reduced to

$$L = \frac{1}{1 - 2\lambda\tau} \quad (14)$$

and

$$B = 2(1 + 2p\bar{p}\tau)G_1(1, 1) - 2p(1-2p)\tau G_1(1, 0) + \frac{2\lambda}{1-2\lambda\tau}\left[\lambda\tau(\tau+1) - \tau + \frac{1-b}{b+a}\right]. \quad (15)$$

The mean queue length at each user at an arbitrary slot boundary is given by

$$\bar{Q} = \frac{B}{2L} = (1 - 2\lambda\tau)[(1 + 2p\bar{p}\tau)G_1(1, 1) - p(1-2p)\tau G_1(1, 0)] + \lambda^2\tau(\tau+1) - \lambda\left(\tau - \frac{1-b}{b+a}\right). \quad (16)$$

Since λ is the mean number of arrivals per user in a slot of length $b + a$, the total throughput of this system is given by $2\lambda/(b + a)$. By Little's result [3], the mean response time D is given by

$$D = \frac{\bar{Q}(b+a)}{\lambda}. \quad (17)$$

Substitutions of the expression for $G_1(1, 0)$ and $G_1(1, 1)$ in (6) and (7) into (16) and some manipulation finally yield

$$D = 1 + a + (b+a) \cdot \frac{\bar{p}^2 + \frac{\lambda p}{2} + \bar{p}\frac{f''(1)}{2\lambda} + p\bar{p}\tau\left[\frac{f''(1)}{2\lambda} + \lambda\left(\tau + \frac{3}{2}\right)\right]}{p\bar{p}(1 - 2\lambda\tau) - \lambda}. \quad (18)$$

In Fig. 2, we show the mean packet delay for given values of throughput, each being optimized with respect to p, in the case of Poisson arrivals for which $f(z) = \exp[\lambda(z-1)]$.

For comparison, we also show the mean response time in a perfect scheduling system (i.e., an $M/D/1$ queue with arrival rate 2λ and service time $1 + a$). This is a plot of

$$D=(1+a) \cdot \frac{1-\lambda(1+a)}{1-2\lambda(1+a)} \tag{19}$$

against the total throughput of 2λ. An interesting observation in Fig. 2 is that D for $b = 0.75$ is mostly greater than D for $b = 1$. This is because when $a = 0.1$ and $b = 0.75$, 71 percent of the second slot in every successful transmission is wasted (note that $(1 + a)/(b + a) = 1.294$ is 0.71 short of the next integer 2). The closeness of the curves for $b = 0.25$ and $b = 0.5$ can be explained similarly.

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A Simulation Study of Clock Recovery in QPSK and 9QPRS Systems

N. A. D'ANDREA AND U. MENGALI

***Abstract*—Computer simulation is employed to assess jitter performance of a clock recovery circuit as a function of the characteristics of the rectifier being used. Several types of rectifiers are compared, some operating at baseband, others at intermediate frequency (IF).**

It is shown that the best choice between them depends both on the modulation format and on the excess bandwidth factor of the pulse spectrum. In QPSK systems, fourth-law rectifiers outperform the others for rolloff factors up to 0.2 while, for higher values, baseband absolute-value rectifiers are preferable. In the case of 9QPRS, baseband absolute-value rectifiers provide jitter reductions of one order of magnitude at high signal-to-noise ratios.

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I. INTRODUCTION

In synchronous pulse amplitude modulation, clock information is necessary at the receiver to detect the transmitted symbols correctly. Clock recovery is a crucial function in systems operating over narrow-band channels, because even small timing errors can greatly degrade the overall performance. A popular method of clock extraction consists in passing the incoming signal (either at IF or at baseband) into a zero-memory device with an even nonlinearity (for short, a rectifier) and then feeding the resulting waveform to a phase-locked loop or to a bandpass filter tuned to the pulse repetition frequency [1]-[8]. Many forms of nonlinearities may be used for this purpose. The most common are the square-law rectifier (SLR) and the absoulte-value rectifier (AVR), but other types have been considered [6].

Clock recovery with SLR followed by resonant circuit has been thoroughly analyzed in [3]. Other things being equal, its behavior depends on the excess bandwidth of the driving pulses in such a way that performance is satisfactory for medium and large values of rolloff factor α, but it becomes poor as α decreases. In the extreme case of minimum bandwidth Nyquist pulses ($\alpha = 0$), this method of clock recovery fails.

Therefore, when dealing with strongly band-limited pulses, nonlinearities other than square-law are required. Unfortunately, clock circuits implemented with non-square-law devices are hardly tractable mathematically, and in fact their performance is mainly known from simulations. References [2] and [4] give results for timing extractors equipped with SLR's or with AVR's. It appears that AVR's hold a substantial advantage over SLR's. A fourth-law rectifier (FLR) has been suggested in [6] in place of an SLR for applications with rolloff as low as 0.12, as in the Bell System 209 data set. Analysis shows that in these conditions an FLR performs well, while an SLR does not.

However useful the above results are, they still leave room for further questions. For example, what is the best choice between SLR, AVR, and FLR when the rolloff varies between zero and unity? Is this choice independent of the signal-to-noise ratio? For fixed rectifier and rolloff, how does the jitter vary as a function of the bandwidth of the clock filter?

In this paper we address these problems by means of computer simulation. Our study concentrates on timing recovery in quaternary phase-shift keyed (QPSK) systems and on nine-state quadrature partial response systems (9QPRS).

II. SYSTEM MODEL

Fig. 1 shows the block diagram of a clock recovery circuit with IF-rectification. The input consists of signal $s(t)$ plus white Gaussian noise $w(t)$ with double-sided spectral density $N_0/2$. The filter output is written as

$$x(t)=x_I(t) \cos \omega_c t - x_Q(t) \sin \omega_c t \tag{1}$$

where ω_c is the IF radian frequency and $x_I(t)$ and $x_Q(t)$ are the in-phase and quadrature components of $x(t)$.

Rectification of $x(t)$ yields the following expressions of the rectifier output (terms centered about multiples of the IF frequency are ignored, as they are rejected by the clock filter):

$$y(t)=\sqrt{x_I^2(t)+x_Q^2(t)} \tag{2}$$

for AVR and

$$y(t)=x_I^2(t)+x_Q^2(t) \tag{3}$$